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ABSTRACT

The recent financial crisis faced by American universities has led to the reevaluation of current forms of financing higher education and has created interest in previously untried methods. One such scheme proposed in the early 1960's--income contingent loans--has received increasing attention; implementation of such loan programs requires either practical experience, which is only now becoming available, or the ability to predict the financial outcome of programs using a systematic modeling approach. The Dresch-Goldberg VTL Simulation Model is designed to fulfill this need, providing flexibility in manipulating the parameters that control the financial outcome of such programs. The structure and computer implementation of the model is described in this article. (Author)

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# THE DRESCH-GOLDBERG VTL SIMULATION MODEL*

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## Abstract

The recent financial crises faced by American universities has led to the re-evaluation of current forms of financing higher education and has created interest in previously untried methods. One such scheme proposed in the early 1960's - income contingent loans - has received increasing attention; implementation of such loan programs requires either practical experience, which is only now becoming available, or the ability to predict the financial outcome of programs using a systematic modeling approach. The Dresch-Goldberg VTL Simulation Model is designed to fulfill this need, providing flexibility in manipulating the parameters which control the financial outcome of such programs. The structure and computer implementation of the model is described in this article.

## I. Introduction

The Dresch-Goldberg VTL (Variable Term Loans) Simulation Model is a two part model constructed to aid in the understanding and design of income contingent loan programs. The Financial Projection Sub-Model traces the relevant future accounts of participants and of the fund to determine the resultant financial status of the fund. The approach in this sub-model is similar to that of other VTL models previously constructed or currently under development.[1] The Analytical Sub-Model and the linkage between the Analytical and the Financial Projection Sub-Models adds two new dimensions to the investigation of income contingent loan programs. The Analytical Sub-Model determines the necessary conditions for a 'viable' loan program, and thus, creates the ability to systematically study and make comparisons between alternative programs. The linkage between the two results in financial projections and profiles which satisfy specified conditions.

To date, the simulation model has been used to evaluate an existing loan program at Yale, the Tuition Postponement Option (TPO), and to

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[1] Other analyses of income contingent loans, all corresponding to the Dresch-Goldberg Financial Projection Sub-Model, include: Karl Shell et al., "The Educational Opportunity Bank: An Economic Analysis of a Contingent Repayment Loan for Higher Education," National Tax Journal, Vol.XXI, No.1 (March 1968); Gail C.A. Cook and David A.A. Stagger, Student Financial Assistance Programs, A Report to the Ontario Committee on Student Awards, (1969), and Morris A. Cohen, MCSSAP (Monte Carlo Simulation of Student Assistance Plans), A User's Manual (January 1971), Institute for the Quantitative Analysis of Social and Public Policy, University of Toronto, Toronto, Canada; Andre Daniere, "The Benefits and Costs of Alternative Federal Programs of Financial Aid to College Students," in The Economics and Financing of Higher Education in the United States, Joint Economic Committee Compendium, Washington, D.C., 1969, and further unpublished work.

evaluate and determine appropriate parameters for incorporation in legislative proposals for the states of Ohio, Illinois and Oregon.

## II. Financial Projection Sub-Model

A basic VTL loan program distributes loans from a fund to students. The fund obtains the capital for the loans by borrowing, paying interest at a rate RB. The recipients are further distributed across education levels. Amounts borrowed (Z matrix) can then be distributed across education levels and an elapsed time (l the lag to repayment) before the student is through with school and thus able to begin repaying. Earning borrowers are finally classified into predefined wage profile groups (Y matrix) in proportions (Q matrix) based on the expected returns to different levels of education. Thus the amounts loaned (AN matrix) to students can also be allocated to these predefined wage profile groups.[2] Since repayment is income contingent it is now possible to treat each group as a unit which makes payments. Each wage group (j is the group classifier) makes payments at a constant proportion (tax rate T) of its yearly earnings and in proportion to the total amounts borrowed ($AN_{1j}$ element of matrix). These payments are credited to the income group's account until it has repaid its debt (in year $NA_j$). The debt just before initiation of repayment is normally computed by adding interest charges at a special lag year interest rate (RL). Interest on the

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[2] See Stephen P. Dresch and Robert D. Goldberg, "Variable Term Loans for Higher Education - Analytics and Empirics," Annals of Economic and Social Measurement, Vol.1, No.1 (January 1972), for details on these distributions.

remaining debt after initiation of repayment is charged at a fixed interest rate (R), not necessarily equal to the lag year rate. If after a specified period (N years) the loan is not fully repaid the debt is automatically cancelled. In order that the fund does not incur a net loss on its total loans, the loan program is designed so that those who have higher incomes repay a higher total amount (discounted to a base comparison year), thus subsidizing those who could not repay their loans. The fund is designed to make a specified profit, for example to break even with a zero profit.

In order to determine whether or not the fund does make the specified profit, an accounting of its yearly costs and revenues must be kept. There are two basic costs to the fund: 1) the yearly interest charges (rate RB) paid on the balance of debt owed to the fund financier, and 2) the administrative costs (ADCST expressed in $/$1000 borrowed) incurred in operating the loan program. The revenues are the sum of payments (TT) for all income groups in a single year; if, however, a certain percentage of that group dies or defaults on payments (SURV vector, representing the proportion who actually pay) then the revenues are reduced by that proportion (resulting in payments of TT*SURV).[3] After accumulating the original debt, the costs and the revenues for a total of N + LL years (LL is the maximum number of lag years), the final balance will then indicate whether the fund has fulfilled its financial aims.

In fact, the Financial Projection Sub-Model follows the above

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[3]The asterisk (*) denotes multiplication.

procedure exactly.

## III. Analytic Sub-Model

Without previous experience or guidelines it will be extremely difficult to set all the parameters in the Financial Sub-Model such that a specified profit is achieved. A hit or miss approach occurs by repetitively running out the model and checking the balance, then repeating until a satisfactory value is found. This technique is slow, inefficient and not highly educational. It was with this problem in mind that the Analytical Sub-Model was designed.

The Analytical Sub-Model first creates the student accounts and runs them to completion using some arbitrary tax rate (T assumed), thus determining the completion years (NA vector) for all the income profile groups. A second set of relationships can be developed from the fund account balances. Using the newly derived completion years, the costs are determined and the ratio of revenues to tax rate can be computed (TT/T), i.e. T assumed is not substituted for this second set. Computing the original debt plus costs and dividing by the ratio of revenues to tax rate yields a solution for a new tax rate. This rate is checked against the assumed rate applied in the student account segment and if they are within specified limits the solution is accepted. If not, the newly derived tax rate is substituted for T assumed and the procedure is repeated again until an acceptable value is found.

The Analytic procedure is efficient and can be used to generate

a system of solutions (spectral solutions) by varying R or N or any of the other inputs. It can be used as a heuristic tool in analyzing the effects of the different factors on specified profit loan programs.[4]

The Analytical Sub-Model also computes the average repayment period, the degree of internal subsidy, and the effective contributions to subsidy by different income profile classes (see primary output table, Table IV). Furthermore, it can be used to determine a simple tax rate to be used in the Financial Projection Sub-Model.

## IV. Contrasting the Operation of Financial and Analytical Sub-Models

The Financial Projection Sub-Model creates the payment streams for a maximum of 120 groups (8 lag years by 15 income profiles) and displays the accounts upon request (see primary output table). The Financial Projection Sub-Model can generate 62 years of recurrent lending terminating the account after the payments are completed in year 100. At termination, it prints the account balance sheet for the fund (see primary output table).

The Analytical Sub-Model aggregates over lag year profiles to form accounts for 15 income profiles; thus a total of 15 payment streams are required. Initial interest charges are made on the basis of lag years and then added to the student and fund accounts, which are kept in units of amounts originally borrowed. This more aggregated approach differs from the Financial Projection Sub-Model, which includes 120

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[4] Such as has been done in Dresch and Goldberg, ibid.

payment streams. These simplifications are necessary to reduce the amount of computer space occupied by the program, thus compensating for the increase in size caused by the additional calculations necessary for the functioning of the analytic approach. Also, since the analytic approach uses a recursive technique execution time would be considerably greater if these initial aggregations were not made.

The aggregation procedure produces slight incongruities between the sub-models. Combining across lag years in the analytical approach creates a common origin at the year of initiation of repayment, compared to a common origin at the year of original borrowing, for the financial approach. If the lag year interest rate (RL) is zero then the two approaches should yield equivalent results. For non-zero values of RL, differences of about one to two percent accrue in the fund balances, depending on RL, the distribution of amounts borrowed, and the other parameters of the program.

The Analytical and Financial Sub-Models operate under different assumptions regarding inflation. The Analytical Sub-Model 'believes' the world is a non-inflationary one and utilizes interest rates expressed in real terms; accounting is discounted to year of initiation of repayments. Converting from a real world to an inflationary one can be accomplished simply by adding the rate of inflation (RF) to the real rate (RR), i.e. the nominal rate (RN) becomes RN = RR + RF. This is a good approximation for small R's since $(1 + RR) * (1 + RF) \approx (1 + RR + RF)$. The Financial Projection Sub-Model incorporates the rate of inflation and expresses accounts in current year dollars; this

makes no difference if the accounts are discounted back to the year of borrowing. The different approaches simply reflect the different objectives of the two sub-models: loan program design in the case of Analytical Sub-Model and financial projection and planning in the case of the Financial Projection Sub-Model.

V. Additional Features of the Sub-Models

Three additional features are incorporated in both sub-models: 1) A multiple of principle factor (EF), 2) minimum payments, and 3) maximum payments. The multiple of principle is applied to the student account at the time of borrowing, and immediately multiplies the debt by this factor. However, the tax payments are still based upon the original amounts borrowed. The Yale Plan, currently in existence, incorporates such a feature.

Minimum payments are incorporated if it is desired to at least return the original principle of the loan or some other pre-determined amount. The yearly payment to be applied can be computed by the program, or it can be set as an exogenous variable. The payment can be made constant in real dollars, in which case it would be increasing in inflated dollars; or, it can be made constant in inflated dollars, in which case it would be decreasing in real dollars. Once the payment has been determined it must be put in either real discounted terms for the Analytical Sub-Model or nominal current dollar terms for the Financial Sub-Model. If the payment is constant in nominal terms and

used as input for the Analytical Sub-Model, it must be adjusted by a factor (involving the rate of inflation and the distribution in time of the amounts borrowed) converting from 'base year' of borrowing in the Financial Projection Sub-Model to the 'base year' of repayment initiation in the Analytical Sub-Model.

Maximum payments provide the most complex options and can either be constant or graduated and can be specified in either real or nominal terms. The constant maximum payment operates just as the constant minimum payment.

The graduated payment, as specified, can take on two functional forms: 1) linear, and 2) exponential. If the functional form is linear in real terms, its form can not be linear in nominal terms, and similarly for the linear case in nominal terms. Again payments must be converted from nominal to real, or vice versa depending on the base. The graduated payment in exponential terms is a simple case (being exponential in both real and nominal terms) and payments are computed by using either the real income growth rate or the inflated income growth rate.

In all cases the option of basing the payments on the amount borrowed or amount owed at initiation of repayment is available. The linearly or exponentially graduated maximum payments are computed to just discharge a conventional loan over a specified period (not necessarily equal to the maximum term). Therefore, when the maximum payment is internally computed, application to the amount owed at repayment initiation is logically indicated, although not required by the computer program.

The format of the model permits a conventional loan program with a

fixed payment schedule to be submitted by the Financial Sub-Model. Thus, the model can simulate a multitude of loan programs from the simplest to the very complex.

## VI. The Model in Mathematical Notation

For those interested in a more concise mathematical description of the basic model, two sets of relationships (A and B) are offered. The symbols used are those introduced in the previous pages. These relationships determine the structure of the model; however, due to the complexity of the computer model, no attempt is made to show the correspondence of these with the logical flow within the computer model.

The number of years a student is making repayments is determined by the relationships:

$$\text{A)}\quad (\text{Balance Owed})_{1j} = EF * AN_{1j} * (1 + RL)^{1} - \sum_{i=1}^{NA_{1j}} P_{1ji}$$

where

$(\text{Balance Owed})_{1j} = 0$ and $NA_{1j} < N$ (total repayment made before end of payment period limit is reached)

or

$(\text{Balance Owed})_{1j} > 0$ and $NA_{1j} = N$ (end of payment period reached before total repayment)

and

$$P_{1ji} = \begin{cases} MIN_i * (1 + R)^{-i} * AN_{1j} & \text{if } MIN_i > T * Y_{ji} * (1 + R)^{-i} \\ MAX_i * (1 + R)^{-i} * AN_{1j} & \text{if } MAX_i < T * Y_{ji} * (1 + R)^{-i} \\ T * AN_{1j} * Y_{ji} * (1 + R)^{-i} & \text{otherwise} \end{cases}$$

The financial outcome of the loan program is then determined in the relationship:

$$\text{B) PROFIT} = -\sum_{1}\sum_{j} AN_{1j} * (1 + RB)^{1} - \sum_{l=1}^{LL}\sum_{j}\left[\sum_{k=1}^{1} ADCST * AN_{1j} * (1 + RB)^{k} + \sum_{i=1}^{NA_{1j}} ADCST * AN_{1j}\right] + \sum_{1}\sum_{j}\sum_{i=1}^{NA_{1j}} SURV_{i} * PB_{1ji}$$

where

$$PB_{1ji} = \begin{cases} MIN_{i} * (1 + RB)^{-i} * AN_{1j} & \text{if } MIN_{i} > T * Y_{ji} * (1 + RB)^{-i} \\ MAX_{i} * (1 + RB)^{-i} * AN_{1j} & \text{if } MAX_{i} < T * Y_{ji} * (1 + RB)^{-i} \\ T * AN_{1j} * Y_{ji} * (1 + RB)^{-i} & \text{otherwise} \end{cases}$$

## VII. Computer Implementation of the Model

The computer program is divided into seven routines; one main initialization routine and six subroutines. Two subroutines apply specifically to the Analytical Sub-Model (TAXR, SMRY) and two specifically to the Financial Projection Sub-Model (INDPY, FUNDB). One subroutine (PMNMX) handles all computations for minimum and/or maximum payment inclusion; and one subroutine (POLCY) receives policy variables, control variables and passes control to the appropriate segments.

Descriptions of the subroutines are contained in Table I. The overall structure of the program is shown in the flow diagram in Figure I.

Input (Table II) for the program is of two types: 1) input relating to participants, and 2) input specifying the details of the loan program. The first set of inputs is received by the main initialization routine (INIT) and prepared for input into the operational segments of the model. The second set of inputs (Table III) is entered as a control vector (V), corresponding to a vector of symbolic names (NAMES), which define the policy and control parameters for the particular loan program to be investigated. After all computations are completed under control of this vector the program returns to receive a new vector V which will guide it if any further projections or analyses are desired.

The program output is outlined in Table IV. It can be divided into three types: 1) analytical output, 2) current dollar student profile accounts, and 3) current dollar financial profile accounts. Using these outputs the policy maker or researcher can make decisions regarding alternative programs.

The computer program was originally designed for the IBM 1130 disk computing system. Since the IBM 1130 is a small system (8K) by present standards, it was necessary to carefully design the programs for maximum efficiency and a minimum of storage space. Operationally, the 1130 version of the model uses three levels of overlays (called LINKS[4] in

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[4]LINK in 1130 terminology means a certain defined set of instructions (i.e. subroutines) are brought into the computer core from a disk storage area. When this set of instructions is completed an additional set contained in disk storage replaces the first, and so on. Control can be passed back and forth between these sets, if the program is designed in this manner.

1130 jargon). The first level contains the main routine INIT; the second level contains POLCY and PMNMX; and the third level either the subroutines for the Analytical or Financial Projection Sub-Model, depending upon which one is being executed, and PMNMX.

The program implementation on the IBM 1130 has provided an excellent interactive tool which could be used by the researcher to 'play games' with alternative programs. By inputing the policy and control vector (V) through the keyboard, responsive decisions could be made by the user and further loan programs generated. This procedure is similar to those available to time sharing users.

The program has recently been converted to and implemented on an IBM 360/67 CMS time sharing system. The design is the same as the 1130 version, except that the need for overlays has been eliminated. The program now operates in about 130K of core; however, further modifications and additions can now be made without concern for space. The conversion has also reduced execution time considerably, although operating costs may not have been affected.

## VIII. An Example of a Model Simulation

**Figures II, III and IV show an example of a session to determine the characteristics of a government loan program involving no greater costs than the existing GLP (Guaranteed Loan Program). The particular income contingent loan program would incorporate an income exclusion of $4000 (real income), i.e. payments would not be made on the first $4000**

of income. The program is based on a 7% interest cost for financing and an 8 1/2% rate charged to the student account.

Figure II shows terminal input and program response for entry of appropriate initial values and control parameters for the loan program. The Model is asked to find an appropriate tax rate and pass it to the Financial Model, which will generate student payment streams and the yearly fund accounts. The program determines a suitable tax rate and types it out, along with the internal subsidy, the average repayment period, the payment period and proportion repayed of original amount borrowed for individual income groups. The Analytic Solution tax rate is then adjusted for minor deviations due to internal computations and the rate of inflation. This solution tax rate is printed along with the amounts owed by income groups with a lag year of one (a negative amount indicates the excess over $1000 paid), the fund balance, administrative cost and total revenue. After this the program is again ready to receive new control information for further executions. (The Government is assumed to contribute the services for administering the fund, thus by providing this external subsidy the need to regain administrative costs from the loan recipients is eliminated, i.e. administrative cost is zero.)

Figure III shows the payment stream for students in income group 11 an lag year one. Due to the lag, payments run to 16 years for a 15 year loan; for students with higher lag years payments would run longer. The columns contain the amounts owed at 8 1/2% and 7% interest, the payment made, whether that payment was the maximum permitted, the

year's income (less $4000 and including inflation), and the administrative cost.

Figure IV shows the payment streams and balances for the fund as a whole. The amounts are based on a total of $1000 loaned; also, since there are eight lag years, payments for a 15 year loan extend to 23 years. The program was specified to make a zero profit; however, it actually returns $17.66 above what it lent. This difference is acceptable by our standards; in fact, we designed the Model so that the outcome will always be slightly more in the black.

TABLE I - SUMMARY DESCRIPTIONS OF ROUTINES

INIT (MAIN ENTRY PROGRAM): Initializes Constants; reads data: constructs income matrix, initial amounts loaned and other vectors and matrices

CALLS POLCY

POLCY (SUBROUTINE): Receives exogenous policy variables and control information

CALLS PMNMX
1. if minimum payments are to be incorporated
2. if maximum payments are to be incorporated

CALLS ANALYTICAL SUB-MODEL
1. if spectrum of solutions for iso-profit programs is desired
2. if tax rate is to be computed for input in FINANCIAL SUB-MODEL

CALLS FINANCIAL SUB-MODEL
1. if all exogenous variables are specified
2. after tax rate has been computed by ANALYTICAL SUB-MODEL

PMNMX (SUBROUTINE):
A. if called by POLCY: Calibrated parameters for calculations of minimum or maximum payments, or both
B. if called by TAXR or INDPY: Calculates minimum or maximum payment

RETURNS to caller

ANALYTIC SUB-MODEL

TAXR (SUBROUTINE): ITERATES to find a tax rate which fulfills a specified profit condition for selected group of participants under given policy assumptions

CALLS PMNMX
1. if minimum payment is to be calculated
2. if maximum payment is to be calculated

CALLS SMRY
RETURNS TO POLCY

SMRY (SUBROUTINE): Computes Average Repayment Period, Subsidy, Present value ratios of percentage repayment made by student; prints results

RETURNS TO TAXR

FINANCIAL PROJECTION SUB-MODEL

INDPY (SUBROUTINE): Simulates individual payment streams to accumulate yearly tax payments and administrative costs for repetitive programs up to 100 years. Prints individual accounts when requested.

CALLS PMNMX
1. if minimum payment is to be calculated
2. if maximum payment is to be calculated

CALLS FUNDB
RETURNS TO POLCY

FUNDB (SUBROUTINE): Constructs and Prints Fund Financial Profile

RETURNS TO INDPY

TABLE II

DRESCH-GOLDBERG VTL MODEL PRIMARY INPUTS

| Symbol | A* | F** | Definition |
|---|---|---|---|
| C | A | F | Base pay by income profile group in year zero |
| D | A | F | Yearly rate of increase |
| IE | A | F | Year in which salary levels off |
| | | | Vectors of dimension 16 which are used to construct 15 income profiles |
| Q | A | F | Matrix 3x15 which contains the proportion of members of a particular educational attainment group (total of 3) over income profile classes (total 15) |
| Z | A | F | Matrix 3x8 which distributes borrowing of a particular educational attainment group over their lag to repayment initiation for a single year of lending by the fund |
| SURV | A | F | Vector of dimension 45 which indicates survival rates for 44 successive years (beginning at age 23); the last element represents an expected default rate |
| DEFLT | A | F | The last (45th) element of the SURV vector, i.e., expected default rate |
| NAMES | A | F | Vector of symbols representing the exogenous policy variables and control variables to be initialized before execution |
| V | A | F | Vector of values used to initialize exogenous policy variables and control variables named in NAMES |

*Variable specific to Analytic Sub-Model

**Variable specific to Financial Projection Sub-Model

TABLE III

VTL MODEL EXOGENOUS POLICY AND CONTROL VARIABLES

| Symbol | A | F | Values* | Definition |
|---|---|---|---|---|
| PRFT | A | F | | Desired net return (in per cent) for total loans |
| N | A | F | | Maximum number of years of repayment |
| T | | F | | Tax rate applied to income; financial input |
| | A | | | Assumed tax rate; analytic input |
| SPEC | A | | YES | Compute tax rates to meet specification for the range of N's and R's specified |
| | | | NO | Check status for TFOR |
| TFOR | A | F | YES | (Disregard if SPEC=YES) Compute a tax rate to meet specifications and pass to Financial Model for use in execution |
| | | | NO | Use specified tax rate in execution of FINANCIAL MODEL |
| TL | A | | | Level of accuracy for tax rate computation |
| TBOR | A | F | YES | Tax rate is per $1000 initially borrowed |
| | | | NO | Tax rate per $1000 owed at initiation of repayment |
| R | A | F | | Student interest rate after payment initiation |
| RB | A | F | | Fund borrowing rate |
| RF | | F | | Rate of inflation |
| RL | A | F | | Student interest rate before payment initiation |
| RLEQ | A | | YES | During execution of Analytic Model RL kept equal to R as it is changed |
| | | | NO | During execution of Analytic Model RL is always equal to the value specified initially |
| RNOM | A | F | YES | If all interest rates (symbols with prefix R) and G are inputed in nominal terms |
| | | | NO | If all interest rates and G are inputed in real terms |
| RU | A | | | Upper-bound limit on R to be used for spectral solutions |

*Unless otherwise noted, values are numeric.

TABLE III (Continued)

| Symbol | A | F | Values* | Definition |
|---|---|---|---|---|
| RLL | A | | | Lower-bound limit on R to be used for spectral solutions |
| NU | A | | | Upper-bound limit on N to be used for spectral solutions |
| NL | A | | | Lower-bound limit on N to be used for spectral solutions |
| LL | A | F | | Maximum number of lag years from a single borrowing year |
| ADCST | A | F | | Administrative cost (per $1000 borrowed) |
| EF | A | F | | Multiple of principal factor |
| KTL | | F | | Total number of successive years which fund makes loans |
| S | | F | | Expansion factor for scaling output |
| IMORT | A | F | YES | Mortality and/or default statistics to be applied to tax revenues |
| | | | NO | No mortality or default included |
| PCH | | F | YES | Punch Financial Model output |
| | | | NO | Do not punch |
| PMIN | A | F | >0 | Then this is the given value of the minimum payment |
| | | | =0 | Then no minimum payments included |
| | | | <0 | Compute PMIN internally |
| PNNOM | A | F | YES | Value of PMIN given or computed is constant in nominal terms |
| | | | NO | Value of PMIN given or computed is constant in real terms |
| PNBOR | A | F | YES | PMIN is the minimum payment per $1000 initially borrowed |
| | | | NO | PMIN is the minimum payment per $1000 owed at initiation of repayment |

*Unless otherwise noted, values are numeric.

TABLE III (Concluded)

| Symbol | A | F | Values* | Definition |
|---|---|---|---|---|
| PMAX | A | F | >0 | Then maximum payment is constant and equal to this given value |
| | | | =0 | No maximum payment included |
| | | | =-1 | The maximum payment is graduated and is expressed as a linear function |
| | | | =-2 | The maximum payment is graduated and is expressed as an exponential function |
| PXNOM | A | F | YES | Value or functional form of maximum payment expressed in nominal terms |
| | | | NO | Value or functional form of maximum payment expressed in real terms |
| PXBOR | A | F | YES | PMAX is the maximum payment per $1000 initially borrowed |
| | | | NO | PMAX is the maximum payment per $1000 owed at initiation of repayment |
| G | A | F | ≥0 | G is the value of the income growth rate in real or nominal terms as specified by RNOM |
| | | | <0 | G is to be computed internally |
| RG | A | F | | Differential increase in R to constrain maximum payment to specified interest rate |
| NH | A | F | | The number of years in which graduated payments are to be made |
| MNEMX | | F | YES | Minimum payment is constrained to equal maximum |
| | | | NO | Constraint is not applied |
| END | A | F | YES | Terminate execution of VTL model |
| | | | NO | Continue execution of VTL model |

*Unless otherwise noted, values are numeric.

TABLE IV

DRESCH-GOLDBERG VTL MODEL PRIMARY OUTPUTS

| Symbol | A | F | Definition |
|---|---|---|---|
| NAMES | A | F | Same as described in inputs |
| V | A | F | Current values of exogenous policy variables |
| K | | F | Elapsed time (in years) VTL program has been in operation (including lag years) |
| T | A | F | Tax rate (per $1000 borrowed) derived by Analytical Sub-Model and/or employed in Financial Sub-Model |
| The following is specific to the Analytic Sub-Model | | | |
| SUB | A | | Value representing the amount of internal subsidy redistribution caused by currently derived VTL loan program |
| XM | A | | Average repayment period, computed by weighting by number in each income profile class |
| NA | A | | Vector of termination years for participants in each income profile class |
| P | A | | Vector of percentage repayment made by 15 income profiles |
| The Following Output is Specific to Individual Student Financial Profiles | | | |
| APN | | F | Vector of yearly balances owed by students computed from tax payments and interest charges made to the students account (normalized to a $1000 unit of borrowing) |
| APFN | | F | Vector of yearly balances of fund debt by student, computed from tax payments, interest charges at the fund borrowing rate RB and administrative costs (normalized to a $1000 unit of borrowing) |
| TSN | | F | Vector of yearly tax payments to fund made by student (normalized to a $1000 unit of borrowing) |

TABLE IV (Concluded)

The Following Output is Specific to Individual Student Financial Profiles

| Symbol | A | F | Definition |
|---|---|---|---|
| YP | | F | Vector of yearly incomes (adjusted by the rate of inflation |
| ACN | | F | Vector of yearly costs (per $1000 unit of borrowing) of administering fund (adjusted by the rate of inflation) |
| APFND | | F | Final amount of debt accrued to a particular account of a student participant discounted to initial lending year |

The Following Output is Specific to the Fund Financial Profile

| | | | |
|---|---|---|---|
| NLNS | | F | Vector of yearly new lending by fund |
| DT | | F | Vector of yearly balances of fund debt including interest costs, new lending, administrative costs and tax payments from student accounts |
| NTCHG | | F | Vector of yearly net changes in fund's financial status, a result of yearly costs and revenues |
| ID | | F | Vector of total yearly interest costs to fund on outstanding balance of fund debt |
| ACOST | | F | Vector of yearly administrative costs |
| TCST | | F | Vector of yearly accounting of accumulated costs including interest charges, administrative costs and new lending |
| TT | | F | Vector of total yearly tax payments received by fund |
| TREV | | F | Vector of yearly accounting of accumulated revenues received from tax payments. Including revenues received by investing them at the fund rate of interest |
| DTD | | F | Final amount of debt accrued to fund at its termination, discounted to the initial creation year |
| ACSTD | | F | Total cost in administering fund discounted to initial creation year |
| TRVD | | F | Total revenues accumulated by fund at its termination discounted to initial creation year |

FIGURE I

FLOW OF CONTROL BETWEEN SUBROUTINES

(LINES and ARROWS indicate paths along which control is passed. DOTS indicate intersections of paths, at these points control can be maintained by host subroutine or passed along one of the junctured paths.)

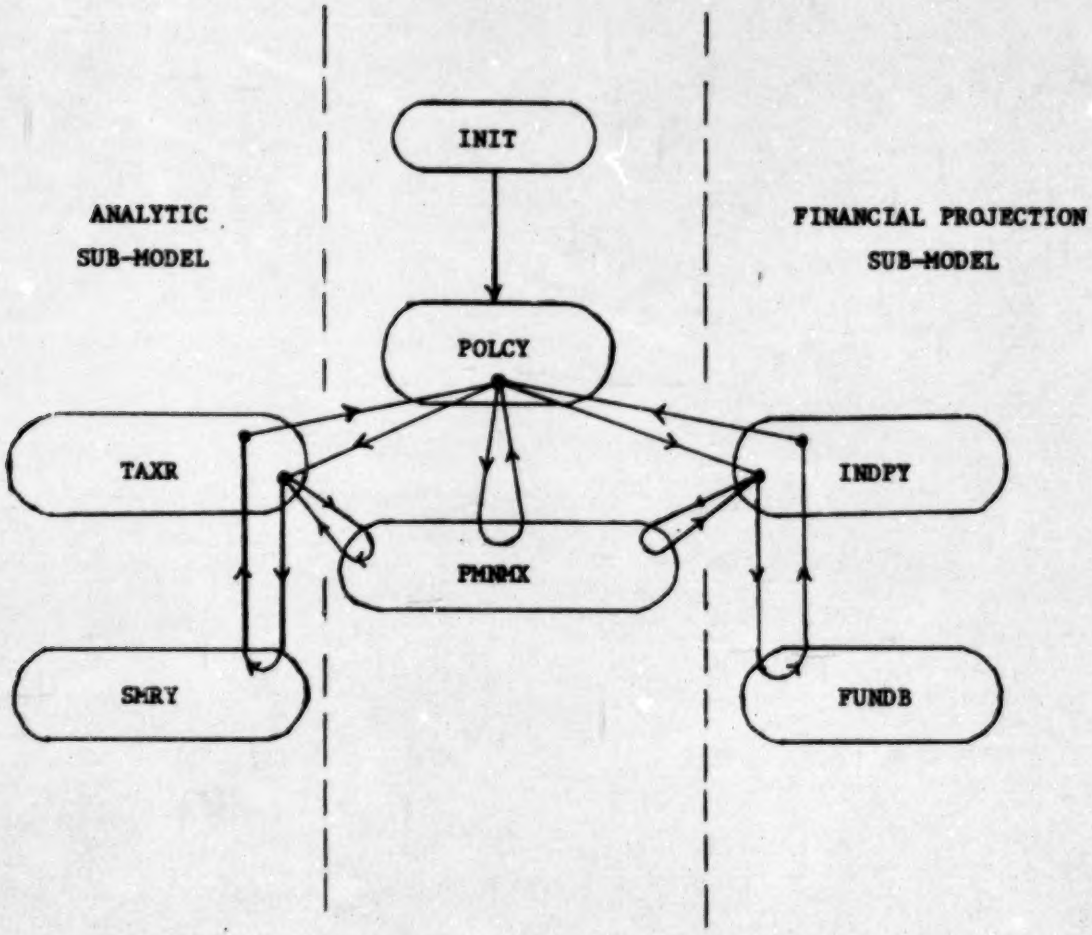

**FIGURE II**

**TERMINAL INPUT AND PROGRAM RESPONSE**

**(User input is typed in lower case letter-ing and program response in upper case.)**

```
tbor
no
t
0.02
pmax
-1.0
r
-1.0
$
N          15 !  NU         35 !  NL         20 !  LL          8 !  KTL         1 !  NH         15 !
T     0.02000 !  TL    0.00001 !  R     0.08500 !  RL    0.08500 !  RB    0.07000 !  RF    0.04000 !
RU    0.10000 !  RLL   0.06000 !  RG        0.0 !  ADCS      0.0 !  EF    1.00000 !  S     1.00000 !
PMIN      0.0 !  PMAX -1.00000 !  G    -1.00000 !  PRFT      0.0 !
SPEC       NO !  TFOR       YES !  TBOR       NO !  RLEQ       YES !  RNOM       YES !  SMOR       NO !
PCH        NO !  END        NO !  PNNO       YES !  PNBO       YES !  PXNO       YES !  PXBO       NO !
MMEM       NO !
ENTER YES TO EXECUTE
      NO TO CHANGE VALUES
      NEW TO READ NEW Z MATRIX


yes
SOLUTION IS T =   0.03593,  FOR R=0.085  AND N= 15
    MIN PAYMENT =  0.0       PNNO =YES
    LINEAR GRAD PAYMENT FUNCTION = 40.11 +  12.33*I      PXNO =YES      I IS THE PAYMENT YEAR

SUBSIDY =   0.12935     AV REPAY PERIOD =  15.00
GROUP       1     2     3     4     5     6     7     8     9    10    11    12    13    14    15
NA(J) =    15    15    15    15    15    15    15    15    15    15    15    15    15    15    15
P(J) =  1.169 1.180 1.166 1.169 1.164 1.164 1.149 1.152 1.152 1.180 1.106 1.137 0.371 0.364 0.000



SOLUTION TAX RATE IS  T =   0.03808

DISCOUNTED VAL OF FINAL AMT OWED AT RB FOR INCOME GROUPS--
     1       2       3       4       5       6       7       8       9      10      11      12      13      14      15
-139.96 -139.96 -139.96 -139.96 -139.96 -139.96 -139.96 -139.96 -139.96 -139.96 -106.27 -139.96  601.30  597.94 1000.00
    MIN PAYMENT =  0.0       PNNO =YES
    LINEAR GRAD PAYMENT FUNCTION = 40.11 +  12.33*I      PXNO =YES      I IS THE PAYMENT YEAR

NEW LOANS OF  $  1000. EACH YEAR FOR A PERIOD OF   1 YEARS

        -17.659  BAL DISCOUNTED TO YEAR OF BORROWING

          0.0    ADMIN COST DISCOUNTED TO YEAR OF BOR

       1017.644  TOT REV DISCOUNTED TO YEAR OF BOR
```

FIGURE III

A STUDENT ACCOUNT BALANCE SHEET

1INCOME GROUP 11 R = 0.085 RL = 0.085 RB = 0.070 RF = 0.040 T = 0.0381 N = 15

INITIAL AMOUNT BORROWED IS $ 1000.00 -- LAG TO REPAYMENT IS 1 YEAR
BORROWING YEAR IS 0

| YEAR | OWED AT R | RB | PAYMENT | INCOME | ADMIN COST |
|---|---|---|---|---|---|
| 1 | 1085.00 | 1070.00 | 0.00 | 0.0 | 0.00 |
| 2 | 1136.82 | 1104.50 | 40.41 | 0.978 | 0.00 |
| 3 | 1176.37 | 1124.74 | 57.08 | 1.381 | 0.00 |
| 4 | 1201.34 | 1128.44 | 75.03 | 1.816 | 0.00 |
| 5 | 1209.14 | 1113.12 | 94.32 | 2.282 | 0.00 |
| 6 | 1201.51 | 1080.63 | 110.41 MAX | 2.784 | 0.00 |
| 7 | 1179.86 | 1032.50 | 123.78 MAX | 3.321 | 0.00 |
| 8 | 1142.99 | 967.62 | 137.16 MAX | 3.898 | 0.00 |
| 9 | 1089.61 | 884.81 | 150.54 MAX | 4.515 | 0.00 |
| 10 | 1018.31 | 782.84 | 163.91 MAX | 5.175 | 0.00 |
| 11 | 927.58 | 660.35 | 177.29 MAX | 5.881 | 0.00 |
| 12 | 815.76 | 515.92 | 190.67 MAX | 6.635 | 0.00 |
| 13 | 681.06 | 347.99 | 204.04 MAX | 7.439 | 0.00 |
| 14 | 521.54 | 154.93 | 217.42 MAX | 8.298 | 0.00 |
| 15 | 335.07 | -65.02 | 230.80 MAX | 9.214 | 0.00 |
| 16 | 119.38 | -313.74 | 244.17 MAX | 10.189 | 0.00 |

-106.27 DIS VALUE OF FINAL AMT OWED AT RB

FIGURE IV

FUND BALANCE SHEET

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| I | | | | | | | | | | | |
| N | 15 | NU | 35 | NL | 20 | LL | 8 | KTL | 1 | NH | 15 |
| T | 0.03808 | TL | 0.00001 | R | 0.08500 | RL | 0.08500 | RB | 0.07000 | RF | 0.04000 |
| RU | 0.10000 | RLL | 0.06000 | RG | 0.0 | ADCS | 0.0 | EF | 1.00000 | S | 1.00000 |
| PMIN | 0.0 | PMAX | 0.42513 | G | 0.10965 | PRFT | 0.0 | | | | |
| SPEC | NO | TFOR | YES | TBOR | NO | RLEQ | YES | RNOM | YES | SMOR | NO |
| PCH | NO | END | NO | PNNO | YES | PNBO | YES | PXNO | YES | PXBO | NO |
| MNEM | NO | | | | | | | | | | |

MIN PAYMENT = 0.0 PNNO =YES

LINEAR GRAD PAYMENT FUNCTION = 40.11 + 12.33*I PXNO =YES I IS THE PAYMENT YEAR

| YEAR | NEW LOANS | FUND BALANCE | NET CHANGE | * | INTEREST | ADMIN COST | TOTAL COST | * | PAYMENT | TOTAL REVENUE |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1000. | 1000. | | | | | | | | |
| 1 | 0.000 | 1069.990 | 70.000 | * | 70.000 | 0.000 | 1069.989 | * | 0.000 | 0.000 |
| 2 | 0.000 | 1132.482 | 62.493 | * | 74.900 | 0.000 | 1144.888 | * | 12.407 | 12.407 |
| 3 | 0.000 | 1186.080 | 53.599 | * | 79.274 | 0.000 | 1225.030 | * | 25.675 | 38.950 |
| 4 | 0.000 | 1228.089 | 42.010 | * | 83.026 | 0.000 | 1310.781 | * | 41.017 | 82.693 |
| 5 | 0.000 | 1255.054 | 26.965 | * | 85.967 | 0.000 | 1402.535 | * | 59.002 | 147.483 |
| 6 | 0.000 | 1268.036 | 12.982 | * | 87.854 | 0.000 | 1500.712 | * | 74.872 | 232.678 |
| 7 | 0.000 | 1265.704 | -2.331 | * | 88.763 | 0.000 | 1605.761 | * | 91.095 | 340.060 |
| 8 | 0.000 | 1248.795 | -16.908 | * | 88.600 | 0.000 | 1718.164 | * | 105.509 | 469.372 |
| 9 | 0.000 | 1215.904 | -32.890 | * | 87.416 | 0.000 | 1838.435 | * | 120.306 | 622.533 |
| 10 | 0.000 | 1166.452 | -49.451 | * | 85.114 | 0.000 | 1967.124 | * | 134.565 | 800.675 |
| 11 | 0.000 | 1098.675 | -67.776 | * | 81.652 | 0.000 | 2104.822 | * | 149.429 | 1006.150 |
| 12 | 0.000 | 1010.557 | -88.117 | * | 76.908 | 0.000 | 2252.159 | * | 165.025 | 1241.604 |
| 13 | 0.000 | 900.099 | -110.457 | * | 70.739 | 0.000 | 2409.809 | * | 181.197 | 1509.712 |
| 14 | 0.000 | 765.192 | -134.907 | * | 63.007 | 0.000 | 2578.495 | * | 197.915 | 1813.306 |
| 15 | 0.000 | 603.702 | -161.490 | * | 53.564 | 0.000 | 2758.989 | * | 215.054 | 2155.290 |
| 16 | 0.000 | 413.459 | -190.242 | * | 42.260 | 0.000 | 2952.117 | * | 232.502 | 2538.660 |
| 17 | 0.000 | 257.971 | -155.487 | * | 28.943 | 0.000 | 3158.764 | * | 184.430 | 2900.795 |
| 18 | 0.000 | 132.128 | -125.842 | * | 18.058 | 0.000 | 3379.876 | * | 143.901 | 3247.751 |
| 19 | 0.000 | 38.570 | -93.558 | * | 9.249 | 0.000 | 3616.467 | * | 102.808 | 3577.899 |
| 20 | 0.000 | -17.210 | -55.780 | * | 2.700 | 0.000 | 3869.618 | * | 58.481 | 3886.831 |
| 21 | 0.000 | -53.769 | -36.559 | * | -1.204 | 0.000 | 4140.488 | * | 35.355 | 4194.258 |
| 22 | 0.000 | -72.013 | -18.243 | * | -3.763 | 0.000 | 4430.320 | * | 14.481 | 4502.328 |
| 23 | 0.000 | -83.714 | -11.701 | * | -5.040 | 0.000 | 4740.438 | * | 6.661 | 4824.148 |

-17.659 BAL DISCOUNTED TO YEAR OF BORROWING

0.0 ADMIN COST DISCOUNTED TO YEAR OF BOR

1017.644 TOT REV DISCOUNTED TO YEAR OF BOR